Bachelor of Science (B.Sc.) Semester–IV Examination

MATHEMATICS

(M_7 Partial Differential Equations & Calculus of Variation)

Paper—I

Time—Three Hours] [Maximum Marks—60

N.B. :— (1) Solve all the **FIVE** questions.

(2) All questions carry equal marks.

(3) Question 1 to 4 have an alternative. Solve each question in full or its alternative in full.

UNIT–I

1. (A) Find the integral curves of the equations :

$$\frac{dx}{y(x+y)+az} = \frac{dy}{x(x+y)-az} = \frac{dz}{z(x+y)}$$ 6

(B) Verify that the equation :

$$x(y^2-a^2)\,dx + y(x^2-z^2)\,dy - z(y^2-a^2)dz = 0$$

is integrable and solve it. 6

OR

(C) If $\bar{x}$ is a vector such that $\bar{x}.\,\text{curl}\,\bar{x} = 0$ and μ is an arbitrary function of x,y,z then prove that $(\mu\bar{x}).(\text{curl}(\mu\bar{x})) = 0$. 6

(D) Eliminate the arbitrary function f from the equation $f(x^2 + y^2 + z^2 ; z^2 - 2xy) = 0$. 6

UNIT–II

2. (A) If u is a function of x, y and z which satisfies the partial differential equation $(y-z)\frac{\partial u}{\partial x}+(z-x)\frac{\partial u}{\partial y}+(x-y)\frac{\partial u}{\partial z}=0$, show that u contains x, y and z only in combinations of x+y+z and $x^2 + y^2 + z^2$. 6

(B) Find the equation of the integral surface of the differential equation

$2y(z-3)p + (2x-z)q = y(2x-3)$ which passes through the circle $z = 0,\ x^2 + y^2 = 2x$. 6

OR

(C) Show that the equations $xp - yq = x,\ x^2p + q = xz$ are compatible and find their solution. 6

(D) Find a complete integral of the equation $p^2x + q^2 y = z$ by using Charpits method. 6

UNIT–III

3. (A) Solve $t + s + q = 0$, where :

$$t=\frac{\partial^2 z}{\partial yz},\ s=\frac{\partial^2 z}{\partial x \partial y} \text{ and } q=\frac{\partial z}{\partial y}$$ 6

(B) Solve $(D^2 - 6DD' + 9D'^2)z = 12x^2 + 36xy$, where $D \equiv \frac{\partial}{\partial x}$ and $D' \equiv \frac{\partial}{\partial y}$. 6

OR

(C) Solve $(D^2 - 3DD' + 2D'^2)z = \sin(x - 2y)$ where $D \equiv \frac{\partial}{\partial x}$ and $D' \equiv \frac{\partial}{\partial y}$. 6

(D) Solve $x^2 \frac{\partial^2 z}{\partial x^2} + 2xy \frac{\partial^2 z}{\partial x \partial y} + y^2 \frac{\partial^2 z}{\partial y^2} = 0$. 6

UNIT–IV

4. (A) Prove that $Fy - \frac{d}{dx} Fy' = 0$ is the necessary condition for an extremum of a functional $I = I[y(x)] = \int_{x_0}^{x_1} F(x, y, y')\,dx$, where $y(x) \in C^{(1)}[x_0, x_1]$. 6

(B) On which curve the functional

$$V(y(x)) = \int_0^{\Pi} \left[(y')^2 - y^2 + 4y\cos x\right]dx,\ y(0) = 0,\ y(\pi) = 0$$

be extremized ? 6

OR

(C) Find the extremals of the functional

$$I = \int_0^1 [(y'')^2 + 2(y')^2 + y^2]dx,\quad y(0) = y(1) = 0,$$

$$y'(0) = 1,\ y'(1) = \sinh(1).$$ 6

(D) Among the functions of class $c^{(2)}$ satisfying the boundary conditions $y(0) = y(\pi) = 0$, $y'(0) = y'(\pi) = 1$. Find the one which can provide an extremum for the functional.

$$I[y(x)] = \int_0^{\Pi} [16y^2 - (y'')^2 + x^2]dx.$$ 6

Question–V

5. (A) Show that equation $ydx + xdy + 2zdz = 0$ is integrable. 1½

(B) Eliminate the constants a and c from the equation $x^2 + y^2 + (z - c)^2 = a^2$. 1½

(C) Write the Jaccobi's auxiliary equations for $p = (z + qy)^2$, where $p = \frac{\partial z}{\partial x}$ and $q = \frac{\partial z}{\partial y}$. 1½

(D) Find the equations $xp - yq = x$, $x^2p + q = xz$, find $\frac{\partial(f,g)}{\partial(x,p)}$. 1½

(E) Solve $xys = 1$, where $s = \frac{\partial^2 z}{\partial x \partial y}$. 1½

(F) Find PI of $(D^2 - 3DD' + D + 1)z = e^{2x+3y}$, where $D \equiv \frac{\partial}{\partial x}$ and $D' \equiv \frac{\partial}{\partial y}$. 1½

(G) For a functional $I[y(x)] = \int_0^1 [y(x)]^2 dx$ find $I(y_1(x))$ if $y_1(x) = e^x$. 1½

(H) If F is linearly dependent on y' defined by $F = P(x,y) + y'Q(x,y)$, then from Euler's equation show that $\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$ 1½